Compressed Sensing for Elastography in Portable Ultrasound

Bonghun Shin¹, Soo Jeon¹, Jeongwon Ryu², and Hyock Ju Kwon¹

Abstract
Portable ultrasound is recently emerging as a new medical imaging modality featuring high portability, easy connectivity, and real-time on-site diagnostic ability. However, it does not yet provide ultrasound elastography function that enables the diagnosis of malignant lesions using elastic properties. This is mainly due to the limitations of hardware performance and wireless data transfer speed for processing the large amount of data for elastography. Therefore, data transfer reduction is one of the feasible solutions to overcome these limitations. Recently, compressive sensing (CS) theory has been rigorously studied as a means to break the conventional Nyquist sampling rate and thus can significantly decrease the amount of measurement signals without sacrificing signal quality. In this research, we implemented various CS reconstruction frameworks and comparatively evaluated their reconstruction performance for realizing ultrasound elastography function on portable ultrasound. Combinations of three most common model bases (Fourier transform [FT], discrete cosine transform [DCT], and wave atom [WA]) and two reconstruction algorithms (L1 minimization and block sparse Bayesian learning [BSBL]) were considered for CS frameworks. Echoic and elastography phantoms, were developed to evaluate the performance of CS on B-mode images and elastograms. To assess the reconstruction quality, mean absolute error (MAE), signal-to-noise ratio (SNRe), and contrast-to-noise ratio (CNRe) were measured on the B-mode images and elastograms from CS reconstructions. Results suggest that CS reconstruction adopting BSBL algorithm with DCT model basis can yield the best results for all the measures tested, and the maximum data reduction rate for producing readily discernable elastograms is around 60%.

Keywords
compressive sensing, model basis, ℓ₁ minimization, Bayesian learning, elastography, elastograms, portable ultrasound, Doppler, strain estimation

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Introduction

Ultrasound elastography (sonoelastography)\(^1\)\(^2\) is a noninvasive medical imaging modality that describes various elastic attributes of tissue to facilitate the detection of malignant lesions. It is based on palpation principle stating that pathological lesions are normally stiffer than benign tissues; therefore, when they are compressed, strains in a malignant lesion are smaller than those in surrounding tissues. Several methods have been developed to calculate the strains, such as time-delay-based,\(^1\) displacement-gradient-based,\(^2\) and phase-based\(^3\) strain estimators. For example, in the time-delay strain estimation (TDE), strains are usually computed from the time delay generally obtained by cross-correlation of pre- and postcompression radiofrequency (RF) echo signals (Figure 1), that is,

\[
\varepsilon_i = \frac{(t_{ib} - t_{ia}) - (t_{2b} - t_{2a})}{t_{ib} - t_{ia}},
\]

where \(t_{ia}\) and \(t_{ib}\) are the arrival times of the precompression echoes from the two reference windows (proximal and distal), respectively, and \(t_{2a}\) and \(t_{2b}\) are the arrival times of the postcompression echoes from the same windows, respectively.\(^1\) Its typical applications are to detect tumors in the breast\(^3\)\(^4\) and the prostate,\(^5\) to monitor thermal changes and ablation,\(^6\) to assess tendon motion,\(^7\) and to measure the stiffness of muscle and tendon.\(^8\)\(^9\)

Recently, portable ultrasound is emerging as a new ultrasound imaging device, which is considerably smaller and lighter than the conventional console style ultrasound scanners. Its high portability and mobility allow practitioners to make diagnostic and therapeutic decisions on site in real time without having to take the patients out of their environment. This makes portable ultrasound an attractive medical modality particularly for harsh and remote sites.\(^10\)

Despite recent development of portable ultrasound devices capable of offering high image quality and multiple ultrasound modes, none of the current devices provide elastography functionality, mainly due to the limitations of hardware performance and data transfer speed of wireless communication. Note that conventional console style ultrasound devices perform large proportion of computations for elastography using dedicated hardware that is specially designed to process the substantial amount of acquired ultrasound data (i.e., 192 channels of echo data with over 20
MHz sampling rate) and sophisticated image processing. Portable ultrasound devices, meanwhile, cannot call on dedicated hardware for such computation; instead, they have to depend on wireless-connected mobile device or laptop computer for image processing and elastography computation. Although the computing power of portable computers has been increasing rapidly, it is still not comparable with that of dedicated hardware. Furthermore, the data transfer speed via wireless communication is insufficient to deliver the large amount of raw ultrasound RF echo data set that is needed to estimate strain fields and generate elastogram images.

One of the reasons a large amount of measurement data in conventional ultrasound is required is the Shannon–Hartley theorem: the sampling rate must be at least twice the maximum frequency present in the recorded signal (the so-called Nyquist rate). When the Nyquist criterion is not met, that is, sampling rate is less than twice the signal frequency, it is known that a condition called aliasing occurs, which results in distortion of the reconstruction compared with the original signal. In general, most ultrasound devices use around four times faster sampling rate than the minimum requirement of Nyquist rate to generate more accurate and higher resolution ultrasound images. Recently, compressive sensing (CS) theory has been actively studied, as a means to overcome the limitation of the conventional Nyquist rate by leveraging the inherent compressibility of most natural signals to allow recovery from far fewer measurements than the Nyquist rate would suggest. It has been typically exploited for the applications that need large amounts of signal acquisition processes such as dynamic magnetic resonance imaging (MRI) and photoacoustic tomography (PAT). CS allows significant reduction of the measurement data, and thus of time for signal processing and data communication while maintaining output signal quality. Moreover, CS can reduce image artifacts and noise power when using the same number of measurements. Given all the benefits of CS, we hypothesize that CS could be a feasible solution to overcome the limitations of portable ultrasound in realizing elastography function. On the contrary, CS construction imposes a new computational load to the paired computing device (laptop or tablet); however, we also hypothesize that with the increase of computing power of such devices, the benefits of CS outweigh the disadvantages.

Note that although medical imaging may be one of the major areas that can be benefited from CS, the adoption of CS in this field is relatively new. Also, most of the related studies have been focusing on conventional pulse-echo B-mode imaging or suggesting several random sampling strategies, and none of them has attempted to apply CS to elastography, particularly for portable ultrasound where reduction of measurement data that to be transferred through wireless communication is crucial. Therefore, the purpose of this research is to examine the feasibility of CS for elastography and to find the most efficient CS framework for implementing elastography function on portable ultrasound. As the CS framework can also be used for B-mode reconstruction using subsampled RF data for reducing wireless communication data, the performance of the frameworks for reconstruction of B-mode images is also investigated. It needs to be mentioned that the quality of CS reconstruction highly depends on both the reconstruction algorithms and the sparsity of the signal representation. Therefore, this study includes composing various CS frameworks associated with different model bases and reconstruction algorithms and assessing the quality of the B-mode images and elastograms from the RF data subsampled and reconstructed by each framework.

The paper is organized as follows. In the “Background” section, we review the CS theory and the major image reconstruction methods. A newly proposed strain estimation method, so-called simple phase-based algorithm, which is significantly faster than the conventional strain estimators is briefly introduced in this section. In the “Method” section, numerical phantoms developed for the virtual experiments are described, and the various CS frameworks composed of several model bases and reconstruction algorithms are presented. Image quality measures for evaluating the results from each CS framework are also described. In the “Results and Discussion” section, the qualities of the images reconstructed by various CS frameworks are compared, and the feasibility of the CS for implementing elastography function on portable ultrasound is discussed. The last section is devoted to the conclusion of this paper.
Background

Overview of CS Theory

CS enables the reconstruction of a signal $x \in \mathbb{R}^n$ with sparse representations from a small number of physical measurements $y \in \mathbb{R}^m, m < n$. The compressed measurement data $y$ is acquired in the so-called sensing basis $f$; thus, it can be mathematically expressed:

$$y = \Phi x,$$

(2)

where $\Phi$ is an $m \times n$ matrix. Random Gaussian ensemble or Bernoulli matrices are often used as a sensing basis $\Phi$ which is designed such that compressible signals $x$ can be recovered exactly from the compressed data $y$.

As most natural signals have concise representations when expressed in a convenient basis, the natural signals are usually significantly compressible. Consider any signal $x \in \mathbb{R}^n$ that can be represented in some model basis $\Psi$ (where $\Psi$ is an $n \times n$ matrix with $\psi_1, \ldots, \psi_n$ as column), which can be an orthonormal basis, a Fourier transform (FT) basis, or other basis depending on the measurement signal. The sparse representation of signal $x$ is

$$x = \sum_{i=1}^{n} v_i \psi_i = \Psi v,$$

(3)

where $v$ is an $n \times 1$ column vector and $x$ and $v$ are the same representation of a signal with $x$ in the time domain and $v$ in the $\Psi$ domain. In the sparse representation, $v$ has only $k < m < n$ nonzero coefficients (so-called $k$-sparse) and the signal $x$ is a linear combination of just $k$ basis vectors. By combining Equations (2) and (3), the measurements can be written as

$$y = \Phi \Psi v = A^{CS} v,$$

(4)

where $A^{CS}$ is an $m \times n$ matrix obeying the so-called restricted isometry property (RIP). In practice, the physical measurements are often corrupted by noise and the measurements with additive noise are rewritten as

$$y = A^{CS} v + z,$$

(5)

where $z$ is a deterministic or stochastic unknown error term and bounds the amount of noise in the data $||z||_2 \leq \epsilon$. As a solution for finding the optimal values of $v$ in Equation (5), two classes of the optimization algorithms have been mainly employed to reconstruct the optimal values of sparse signal $v$; the first one uses deterministic algorithms including L1 minimization (L1) algorithms, and the other uses stochastic algorithms using the Bayesian learning framework, such as block sparse Bayesian learning (BSBL).

In the L1 approaches, reconstruction can be first performed by solving the following minimization problem, given by

$$P: \hat{v} = \arg \min_{v \in \mathbb{R}^n} \|v\|_1 \text{ subject to } \|y - A^{CS} v\|_2 \leq \epsilon.$$

(6)

In solving Equation (6), a sparse reconstruction algorithm estimates the optimal values of $v$ in Equation (5), and then the signal $x$ can be computed from Equation (3).

On the contrary, the unknown sparse signal $v$ can also be reconstructed by exploiting the principle of Bayesian inference as a stochastic algorithm. In this approach, a priori probability
density functions (PDFs) are associated with each of the unknown variables \( v \), and the Bayes law is used to find the posteriori probability to be maximized, such that

\[
p(v \mid y) \propto p(y \mid v) p(v),
\]

where \( p(y \mid v) \) represents the likelihood and \( p(v) \) contains prior information about the unknown sparse \( v \). Assume that the \( A^{\text{CS}} \) matrix is known and the noise \( z \) is approximated by an additive Gaussian noise with zero mean and unknown variance \( \sigma^2 \). Then the sparse coefficients \( v \) and the noise variance \( \sigma^2 \) are the quantities of CS estimate based on Bayesian framework. The associated Gaussian likelihood model is given by

\[
p(y \mid v, \sigma^2) = \left( \frac{\pi \sigma^2}{2} \right)^{-n} \exp \left( -\frac{1}{\sigma^2} \| y - A^{\text{CS}} v \|^2_2 \right).
\]

By introducing a priori knowledge about the coefficients to be recovered, the sparsity model is modeled as follows:

\[
p_0 (v) \propto \exp \left( -\| v \|_0 \right).
\]

We see that the problem of recovery of the sparse coefficients of \( v \) is transformed into a linear-regression problem using a Bayesian prior constraint.

BSBL algorithms have been proposed to further improve reconstruction performance of wireless electrocardiogram (ECG) applications. By exploring and exploiting the intrablock correlation that correlates the entries in each block, the recovery performance of BSBL was greatly improved and compared with other methods ignoring the intrablock correlation. The BSBL algorithms also have a pruning mechanism in which they use a threshold to prune out irrelevant coefficient. Recently, several BSBL algorithms such as BSBL–Expectation Maximization (BSBL-EM), BSBL–Bound Optimization (BSBL-BO), and BSBL-L1 have been proposed. In this work, BSBL-BO which is known to have balanced performance and speed is selected to show the CS reconstruction performance in generating elastography for portable ultrasound.

**Model Bases**

CS performance strongly depends on signal sparsity representation in the model basis \( \Psi \); however, as the raw RF data in ultrasound show an oscillatory pattern, it is not trivial to find the adequate sparsity representation with any basis. In this study, we considered three types of model basis: wave atom (WA), discrete cosine transform (DCT), and discrete FT.

The WA basis, recently proposed by Demanent and Ying, describes the exact relationship between the directional wavelets and the Gabor transform (a special case of the short-time FT). Consequently, it produces a multiscale transform with frame elements indexed by scale, location, and orientation parameters. The multiscale feature in WA is useful for adapting to arbitrary local directions of oscillatory patterns. Meanwhile, DCT expresses a finite sequence of data points in terms of a sum of cosine functions (real-valued) oscillating at different frequencies, while FT represents scaled-and-shifted complex vectors in the frequency domain.

**Simple Phase-Based Strain Estimator**

The axial strain of a segment that has been deformed along loading direction is defined as

\[
\varepsilon = \frac{\Delta L}{L} = \frac{L - L_0}{L_0},
\]

(10)
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where $\Delta L$ is the difference between the final length $L$ and initial length $L_0$ of the segment.

In elastography, it can be assumed that an ultrasonic transducer transmits waves toward an object moving with an instantaneous velocity $V_2$ and $V_1$, respectively (Figure 2). As a result, the segment length $L_0$ at $t = T_0$ is changed to $L$ at $t = T_0 + T_{PR}$. SPSE = simple phase-based strain estimator.

Now let us consider an axial segment along a single scan line. If the segment is centered at $m$ depth samples with the upper and lower endpoints given by $m_1 = m - \Delta m / 2$ and $m_2 = m + \Delta m / 2$, the axial length of the segment is

$$L_0 = \Delta m \frac{c}{2} T_s,$$

where the tunable parameter $\Delta m$ controls the length of the axial length of the segment. By substituting Equation (13) into Equation (11) and rewriting $V_1$ and $V_2$ using Equation (12), the local axial strain can be simplified as

$$\varepsilon = \frac{T_{PR}}{L_0} (V_2 - V_1),$$

where the corresponding velocity $V_i$ at both endpoints of the segment is evaluated by two-dimensional (2D) autocorrelator$^{21,22}$ and is expressed as

$$\hat{V}_i = \frac{c}{2} \frac{T_s}{T_{PR}} \frac{\text{arg}\{\gamma_i[1,0]\}}{\text{arg}\{\gamma_i[0,1]\}} = \frac{c}{2} \frac{T_s}{T_{PR}} \Gamma_i,$$

where $\text{arg}\{\gamma_i[1,0]\}$ and $\text{arg}\{\gamma_i[0,1]\}$ are the phase of autocorrelation at lags in sampling interval and pulse repetition period, respectively.

Figure 2. Principle of strain estimation in SPSE: ultrasonic transducer transmits waves toward a segment (left). The lower (farthest away from the transducer) and upper endpoints of the segment are moving with an instantaneous velocity $V_2$ and $V_1$, respectively (right). As a result, the segment length $L_0$ at $t = T_0$ is changed to $L$ at $t = T_0 + T_{PR}$. SPSE = simple phase-based strain estimator.
\[ \varepsilon = \frac{\Gamma_2 - \Gamma_1}{\Delta m}, \]  

(14)

where \( \Gamma_1 \) and \( \Gamma_2 \) are the 2D autocorrelation values at both endpoints of the segment.

Note that Equation (14) contains only segment length \( \Delta m \) and the phase angle \( \Gamma \) at the upper and lower endpoints of the segment; thus, it is not affected by sampling intervals along the depth (\( T_s \)) and the frame (\( T_{PR} \)). In portable ultrasound, the sampling interval along the frame (\( T_{PR} \)) refers to the time interval to make a pair of RF data sets from the tissue before and after the physical compression. This time interval (equivalent to pulse repetition period) is also equivalent to data-dumping interval via Wi-Fi network established between portable ultrasound and the paired computing device; therefore, the sampling interval cannot be as uniform or stable as that of console style scanner. However, Equation (14) indicates that although data-dumping interval is not consistent, strain estimation accuracy is not degraded in the simple phase-based strain estimator (SPSE) method.

In this study, CS is applied to portable ultrasound following the procedure as illustrated in Figure 3. First, a pair of original RF data sets are collected and compressed by the portable ultrasound device (Figure 3, above). The undersampled (compressed) RF data sets are then transmitted to a laptop or mobile device through the Wi-Fi network established between them. The laptop computer (or mobile device) recovers the compressed data sets and generates the elastogram using the SPSE method (Figure 3, bottom).

**Method**

**Numerical Phantoms**

Numerical phantoms were developed to perform the virtual ultrasound experiment to evaluate the performance of various CS frameworks on image reconstruction. Two types of numerical phantoms were modeled: echoic and elastography phantoms.

An echoic phantom contains arrays of hyperechoic and hypoechoic inclusions\(^{17}\) to assess the performance of CS on the recovery of B-mode images. Using Field II,\(^{23,24}\) an open-source Matlab-based ultrasound simulation code, RF signals from a numerical phantom of size \( 50 \times 10 \times 55 \text{ mm}^3 \) were simulated. A 192-element linear array probe with the center frequency 3.5 MHz was modeled to generate the regular ultrasound B-mode images. The numerical phantom was composed of a total of 100,000 point scatterers, four hyperechoic, and four hypoechoic inclusions of the diameter of 6 mm. The hyperechoic inclusions mimicked the malignant tumor with round hyperdensities (BiRads 4 or 5), while hypoechoic inclusions simulated benign cysts filled with liquid without any scatterers (BiRads 1 or 2). The standard deviation of the scatterers’ amplitude distribution inside the hyperechoic inclusions was 10 times that of the background. The spatial distribution of the scatterers in the hyperechoic inclusions and the background was modeled as uniform distribution, and the amplitude of these regions followed a zero mean Gaussian distribution, respectively.

An elastography phantom was constructed by combining a finite element analysis (FEA) model and Field II code. Using commercial FEA code (Abaqus/CAE 6.10, Providence, RI, USA) (Figure 4, upper left), a linear elastic phantom of size \( 40 \times 50 \times 10 \text{ mm}^3 \) was modeled to have a stiff cylindrical inclusion (10 mm) in the soft matrix. The FEA model was meshed with approximately 427,000 three-dimensional (3D) quadratic tetrahedron elements and 77,000 nodes. The elastic moduli of the matrix and the inclusion were set to 20 and 100 kPa, respectively, mimicking a carcinoma in breast tissue. Poisson’s ratio of 0.49 was applied to the whole phantom. The vertical movement of the bottom surface of the phantom was constrained while 0.1\% axial compressive...
strain was applied to the top surface. The coordinates of each node were determined and recorded by FEA as the deformation field data sets. Then Field II code was used to add random scatterers to the nodal displacements and generate the corresponding pre- and postdeformation RF signal data (Figure 4, upper center). The amplitudes of the random scatterers were kept constant throughout the phantom; thus, the inclusion could not be detected in the RF signal or in the B-mode image. To simulate the portable ultrasound device, a linear probe having 152 ultrasound elements and 24 active elements was virtually modeled with Field II. The center frequency of the transducer was placed at 3.5 MHz and the sampling rate of RF signals was set to 28 MHz. The speed of sound...
through the phantom was set to 1540 m/s. With this setting, Field II generated 128 simulated RF lines (A-lines) with each line containing 2,589 samples across the phantom depth. Parameters for both echoic and elastography phantoms are listed in Table 1. The SPSE was applied to the RF data sets from CS reconstruction (Figure 4, upper and lower right) and the strain fields were estimated from the reconstructed RF data set (Figure 4, lower center). The differences between the strain estimates and the true strains computed by the FEA model were regarded as estimation errors (Figure 4, lower left).

Table 1. Parameters for Numerical Phantoms.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Echoic Phantom</th>
<th>Elastography Phantom</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phantom size</td>
<td>50 × 10 × 55 mm³</td>
<td>40 × 50 × 10 mm³</td>
</tr>
<tr>
<td>Center frequency</td>
<td>3.5 MHz</td>
<td>3.5 MHz</td>
</tr>
<tr>
<td>Sampling frequency</td>
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<td>28 MHz</td>
</tr>
<tr>
<td>Width</td>
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<td>0.44 mm</td>
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<tr>
<td>Height</td>
<td>5 mm</td>
<td>5 mm</td>
</tr>
<tr>
<td>Kerf</td>
<td>0.022 mm</td>
<td>0.022 mm</td>
</tr>
<tr>
<td>Number of elements</td>
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<td>152</td>
</tr>
<tr>
<td>Transmit elements</td>
<td>64</td>
<td>24</td>
</tr>
<tr>
<td>Receive signals considered</td>
<td>128</td>
<td>128</td>
</tr>
<tr>
<td>Transmit/receive focus</td>
<td>50 mm</td>
<td>50 mm</td>
</tr>
</tbody>
</table>

Figure 4. Schematic of the procedure to construct a virtual elastography phantom and to produce elastogram from the undersampled RF data of the phantom using CS reconstruction. RF = radiofrequency; CS = compressive sensing; FEA = finite element analysis; MAE = mean absolute error; SNRe = signal-to-noise ratio; CNRe = contrast-to-noise ratio; SPSE = simple phase-based strain estimator; IQM = Image Quality Measures.

Model Bases

To find a relevant sparse representation of the raw RF data in ultrasound, reconstruction performance of the CS adopting WA, DCT, and FT model bases were compared. As the elastogram used in this study depicts the axial strain field, each basis function is applied to one-dimensional
(1D) RF signal, and then the measurement signal $x$ is converted to the sparse representation $v$ in the $\psi$ domain as described in Equation (2). For WA model base, the WA package based on the study of Demanet and Ying was employed to conduct the forward and inverse WA transform. For DCT and FT, 1D built-in function sets in Matlab were utilized, with the signal segment size set to 256 for all model bases.

**Reconstruction Algorithms**

The simulated RF data sets produced from the numerical elastography phantom were subsampled by removing 10% to 80% of the original samples using a uniform random law. For example, 70% subsampling rate suggests that 70% of the original samples are removed and only 30% are maintained in the compressed vector $y$. CS reconstruction was then performed on the subsampled RF data by solving the CS minimization problem in Equation (5). Two types of optimization algorithms were adopted: L1 and BSBL. In the L1 experiments using the $\ell_1$-Magic package, the accuracy threshold $\epsilon$, signal segment size, and the number of maximum iteration were set to 0.003, 256, and 50, respectively. BSBL, a recently proposed stochastic-based reconstruction algorithm, was also implemented by using the BSBL-BO package. In the BSBL experiments, the segment length and block size, the accuracy threshold $\epsilon$, and the maximum iteration were set to 256, 12, 1e-8, and 7, respectively. During the experiment, both methods successfully solved the reconstruction problem without any stability issue.

**Image Quality Measures**

The accuracy of the B-mode and elastogram images from CS reconstruction were quantified by comparing them with the images from the original data through the mean absolute error (MAE) given by

$$\text{MAE} = \frac{1}{N} \sum_{i=1}^{N} |I_o - I_r|,$$

where $N$ is the total number of the image data, and $I_o$ and $I_r$ are the intensities of both original and reconstructed images, respectively.

The image quality of elastograms was examined using the signal-to-noise ratio (SNRe) and the contrast-to-noise ratio (CNRe), where $e$ stands for elastography. Specifically, the elastographic SNRe identifying the quantitative measurement of the accuracy and the precision of an elastogram is defined as

$$\text{SNRe} = \frac{m_s}{\sigma_s},$$

where $m_s$ is the mean value of the strain, and $\sigma_s$ is the standard deviation of the measured strain.

The elastographic CNRe is an important parameter to determine the detectability of a stiff lesion in an elastogram and is defined as

$$\text{CNRe} = \frac{2(m_o - m_i)^2}{\sigma_o^2 + \sigma_i^2},$$

where $m_i, m_o, \sigma_i^2, \text{ and } \sigma_o^2$ are the mean values and the variance values for the inside (subscript $i$) and the outside (subscript $o$) of the lesion, respectively.
Results and Discussion

Evaluation of B-Mode Reconstruction

B-mode images produced by various CS frameworks formed by combining one of the two reconstruction algorithms (L1 and BSBL) and one of the three model bases (FT, DCT, and WA), respectively, were evaluated as shown in Figures 5 and 6 to demonstrate the general CS application in medical ultrasound. To compare the quality of reconstructed B-mode images, we selected a 50% subsampling rate in all cases when generating the elastograms.

As for the echoic phantom containing four hyper- and hypoechoic inclusions (Figure 5), both L1 (Figure 5a) and BSBL (Figure 5b) algorithms were able to recover the detailed patterns of the phantom, and their hyper- and hypoechoic inclusions are clearly discernable, except the framework combining L1 with FT basis (L1-FT). Among the B-mode images reconstructed by L1 (Figure 5a), both L1-DCT and L1-WA show comparable image quality and similar MAE values of 0.082. L1-FT produces the lowest image quality with the highest MAE value (0.242). Hyper- and hypoechoic inclusions on L1-FT image are blurred and dispersed, which make them difficult to discern. On the contrary, B-mode images reconstructed by BSBL present better image quality and lower MAE than those by L1, as shown in Figure 5(b). BSBL-DCT produces the lowest MAE of 0.022, while the MAEs of BSBL-FT and BSBL-WA are slightly higher at 0.029 and 0.037, respectively.

B-mode images of the elastography phantom containing a stiff inclusion from various CS frameworks are shown in Figure 6. As the standard deviation of the scatterers’ amplitude distribution is the same as that of background, the inclusion is not visible on the B-mode images. Among the B-mode images reconstructed by L1 (Figure 6a), both L1-DCT and L1-WA are associated with the same level of MAE at 0.066, whereas L1-FT yields the highest MAE of 0.111 with unexpected vertical black patterns appearing on the reconstructed image. Meanwhile, B-mode images reconstructed by the BSBL (Figure 6b) preserve the patterns intact with excellent accordance with the original image. Comparing MAE values associated with the same model bases, BSBL-based frameworks yield much lower values than L1-based ones. Among the BSBL images in Figure 6(b), BSBL-FT is associated with the lowest MAE of 0.017, followed by BSBL-DCT and BSBL-WA with the MAE of 0.022 and 0.034, respectively.

Plots of MAE values for various CS frameworks are presented in Figure 7 as a function of removed data (subsampling rate) from 10% to 80%. Quite consistently, the errors increase with the number of samples removed for all CS frameworks. For the echoic phantom (Figure 7a), the MAE values increase linearly until 50% subsampling for all model bases, and then rapidly rise, except for L1-FT, which shows a linear trend with much higher error than the other two bases. It is also notable that BSBL-based frameworks yield lower MAE values than L1-based ones with little variation between model bases, which is consistent with the trends in B-mode images (Figures 5 and 6). The MAE plots for L1-DCT and L1-WA are almost equivalent, while all BSBL-based plots agree well with each other. In case of the elastography phantom (Figure 7b), the trends of MAE are similar to those of the echoic phantom, apart from L1-FT which is still higher than the others but follows much closer trend than that in echoic phantom. Overall, MAE values associated with BSBL-based frameworks are lower than those of L1-based ones with little variation across model bases.

Evaluation of Elastograms

By applying the SPSE method to the RF data of elastography phantom from CS reconstruction, elastograms are generated to describe the strain fields under compressive deformation. Image quality of the elastograms from various CS frameworks is comparatively investigated.
Figure 5. B-mode images of the echoic phantom containing hyper- and hypoechoic inclusions produced from the original data and the reconstructed data by (a) L1-based and (b) BSBL-based CS reconstruction frameworks, combined with FT, DCT, and WA model bases, respectively. Data were reconstructed using 50% subsampling. BSBL = block sparse Bayesian learning; CS = compressive sensing; FT = Fourier transform; DCT = discrete cosine transform; WA = wave atom; MAE = mean absolute error.
Figure 6. B-mode images of the elastography phantom produced from the original data and the reconstructed data by (a) the L1-based and (b) the BSBL-based CS reconstruction frameworks, combined with FT, DCT, and WA model bases, respectively. Data were reconstructed using 50% subsampling. BSBL = block sparse Bayesian learning; CS = compressive sensing; FT = Fourier transform; DCT = discrete cosine transform; WA = wave atom; MAE = mean absolute error.
Figure 7. MAE plots associated with various CS frameworks as functions of subsampling rate, measured on (a) the echoic phantom and (b) the elastography phantom. MAE = mean absolute error; CS = compressive sensing; FT = Fourier transform; DCT = discrete cosine transform; WA = wave atom; BSBL = block sparse Bayesian learning.

The elastograms from L1-based frameworks for the subsampling rate from 30% to 50% are compared in Figure 8. The elastograms for all bases for 30% subsampling rate preserve the original patterns very well, and the stiff inclusion in the center is clearly discernable. At 40% subsampling rate (Figure 8b), both L1-DCT and L1-WA elastograms still show discernable inclusion and consistent matrix strain which are close to the original image, while degradations in the inclusion
and the matrix start occurring in L1-FT elastogram. When the subsampling rate is increased to 50% (Figure 8c), the shapes of the stiff inclusion for all three bases are hardly discernable and the strain fields in the matrix show inconsistent and locally varying behavior. Overall, L1-DCT and L1-WA preserve the strain patterns of similar quality until 40%, while L1-FT tends to lose the patterns much earlier than the others. From the observation, 40% subsampling rate seems to be the threshold compression ratio to effectively detect the inclusion for the elastograms from L1-based CS frameworks.

Strain values measured along the vertical centerline across the L1-based elastograms are plotted in Figure 9. The strain fields for three bases over the subsampling rate from 30% to 50% are compared with the strains from the FEA as a ground truth. At 30% subsampling rate (Figure 9a), both plots from L1-DCT and L1-WA show good agreement with the FEA strains. For 40% (Figure 9b), L1-DCT and L1-WA still follow the trend of FEA, but the strains start oscillating both in the inclusion and the matrix where strains are regarded as constant. The oscillations in these regions are significantly amplified with further increase of subsampling rate (Figure 9c). Beyond 50%, the strain plots become too noisy to identify the shape of the inclusion, which also indicates that sampling rate around 40% should be the threshold for L1-based CS frameworks.

The elastograms from BSBL-based CS frameworks over the subsampling rate from 50% to 70% are presented in Figure 10. At 50% subsampling rate (Figure 10a), all elastograms preserve the patterns superbly; they are almost equivalent to the original elastogram and

![Figure 8. Elastograms of elastography phantom computed from the original data and from various CS reconstruction frameworks for the subsampling rate of (a) 30%, (b) 40%, and (c) 50%. CS = compressive sensing; FT = Fourier transform; DCT = discrete cosine transform; WA = wave atom.](image-url)
accurately depict strain distribution in the inclusion and the matrix. At 60%, the inclusion is still discernable, regardless of slight strain degradation particularly in BSBL-WA (Figure 10b). Beyond 70% subsampling rate, all elastograms are significantly degraded and the original strain patterns are lost almost completely as shown in Figure 10(c). Qualitative observation suggests that BSBL-DCT elastograms best agree with the original ones, particularly for 50% and 60% subsampling rate.

The strain plots along the vertical centerline across the BSBL-based elastograms are presented in Figure 11. The strain plots for all three bases show excellent agreement with the ground truth (FEA results) for 50% subsampling rate (Figure 11a). With the increase of subsampling rate, reconstructed strain plots start to show oscillating behavior (Figure 11b). Eventually, all strain plots lose the track of the ground truth beyond 70% subsampling rate, as shown in Figure 11(c). It can be summarized that BSBL-based CS reconstruction is highly reliable until 50%, and is reasonably accurate up to 60% subsampling rate, for all three bases tested. Furthermore, the qualities of elastograms from BSBL-based CS frameworks are far less influenced by the model bases than those from L1-based ones.

Figure 9. Strain values measured along the vertical centerline across the elastograms computed from the L1-based CS reconstruction frameworks for the subsampling rate of (a) 30%, (b) 40%, and (c) 50%. The FEA plots are the ground truth. CS = compressive sensing; FEA = finite element analysis; FT = Fourier transform; DCT = discrete cosine transform; WA = wave atom.
Figure 10. Elastograms of the elastography phantom computed from the original data and from the BSBL-based CS reconstruction frameworks for the subsampling rate of (a) 30%, (b) 40%, and (c) 50%. BSBL = block sparse Bayesian learning; CS = compressive sensing; FT = Fourier transform; DCT = discrete cosine transform; WA = wave atom.

Evaluation of Image Quality Measures and Computation Times

Image qualities of the elastograms are evaluated with three image quality measures (MAE, SNRe, and CNRe) to determine the optimal CS scheme for generating the ultrasound elastograms. All the image measures are collected over the subsampling rate from 10% to 80%.

The MAE plots of elastograms (Figure 12) from various CS frameworks are compared with the reference strain error (black solid line) that corresponds to 15% of the applied strain (0.1%). The reference error plays as the error criterion based on the observation that MAE plots rise rapidly once they reach this level. As MAE can be regarded as a monotonic function of subsampling rate, the threshold subsampling rate of each framework is estimated from the intersection between the MAE and the error criterion. Among the MAE plots for L1-based frameworks (dashed lines in Figure 12), the L1-FT yields the highest error level and intersects the error criterion at around 33% subsampling rate, while L1-DCT and L1-WA are slowly increasing until 40% subsampling rate from which they start rising rapidly. Overall, among L1-based frameworks, L1-WA presents the best result until it reaches the error criterion.

All BSBL-based CS frameworks generate similar level of MAE lower than error criterion until 50%, regardless of associated model bases. Threshold subsampling rate is identified to be around 60% for BSBL-WA and BSBL-FT and around 63% for BSBL-DCT. The comparison
between L1- and BSBL-based plots in Figure 12 suggests that BSBL-based CS frameworks yield more reliable results than L1-based ones. Particularly BSBL-DCT yields the lowest error level over the subsampling range tested.

The elastographic SNRe and CNRe identifying the precision and the discernibility of the elastograms are quantified in Figure 13. All SNRe plots in Figure 13(a) present slowly decreasing trend at first but start to drop rapidly with increase of subsampling rate. BSBL-based frameworks yield higher SNRe than L1-based ones across all subsampling range tested. In Figure 13(b), CNRe plots from CS frameworks are almost equivalent to those from original elastogram (meaning excellent discernibility) in low subsampling range; however, they start to drop rapidly with increase of subsampling rate. Overall, both SNRe and CNRe plots from BSBL-based frameworks present higher values than those from L1-based ones over the entire subsampling range. Furthermore, results from BSBL-based frameworks are less influenced by the model bases because its block-wise approach might maximize the signal sparsity of ultrasound echo signal. All the image quality measures (MAE, SNRe, and CNRe) imply that the feasible level of the subsampling rate without significant loss of patterns is 40% for L1-based and 60% for BSBL-based frameworks, respectively.

**Figure 11.** Strain values measured along the vertical centerline across the elastograms computed from the BSBL-based CS reconstruction frameworks for the subsampling rate of (a) 50%, (b) 60%, and (c) 70%. BSBL = block sparse Bayesian learning; CS = compressive sensing; FT = Fourier transform; DCT = discrete cosine transform; WA = wave atom.
Figure 12. MAE of the elastograms as a function of subsampling rate. The error is computed on the elastograms produced from the various CS reconstruction frameworks. Reference error is 15% of the applied strain. MAE = mean absolute error; CS = compressive sensing; FT = Fourier transform; DCT = discrete cosine transform; WA = wave atom; BSBL = block sparse Bayesian learning.

Computation times of the CS reconstruction methods were also measured on a Windows 10 computer (2.3MHz, i7-3670 CPU with 12 GB RAM, ASUS-K55VD) using the in-house developed Matlab code. Overall, L1-WA showed the fastest computation time (16.732 seconds) while L1-FT spent 105.909 seconds due to the calculation of its complex array. On the contrary, BSBL-based frameworks presented relatively similar computation times among different bases. BSBL-WA took only 38.454 seconds, while BSBL-DCT and BSBL-FT recorded 41.864 and 55.479 seconds, respectively. As for the number of average iterations, L1-FT recorded 42.61 iterations, while L1-DCT and L1-WA took 12.95 and 12.73 iterations, respectively. For the BSBL-based frameworks, all methods required around seven iterations. Threshold subsampling rates and the corresponding MAEs, SNRe, CNRe, and computation times for different CS frameworks are summarized in Table 2.

Conclusion

Large amount of ultrasound echo data to be transferred through wireless communication is one of the major limitations in implementing ultrasound elastography function on portable ultrasound. As a means to reduce the size of the measurement data, this paper addresses the feasibility of applying CS method to elastography. As CS reconstruction performance is highly affected by the model basis representing the sparse expansion of the data, as well as reconstruction algorithm to solve the minimization problem, we tested three bases, discrete FT, DCT, and the recently introduced WAs, and two reconstruction algorithms, L1 minimization (L1) and BSBL.

The quality of the reconstructions was quantified using the B-mode and elastogram images of simulated numerical phantoms through three image quality measures, MAE, SNRe, and CNRe,
Table 2. Image Quality Measures, CPU Time, and Average Number of Iteration at the Threshold SR Associated with Various CS Reconstruction Frameworks.

<table>
<thead>
<tr>
<th>CS Reconstruction</th>
<th>CS Model Basis</th>
<th>Threshold SR (%)</th>
<th>MAE</th>
<th>SNRe (dB)</th>
<th>CNRe (dB)</th>
<th>CPU (Seconds)</th>
<th>Average Iteration</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>FT</td>
<td>40</td>
<td>1.899e-4</td>
<td>2.052</td>
<td>34.564</td>
<td>105.91</td>
<td>42.61</td>
</tr>
<tr>
<td>L1</td>
<td>DCT</td>
<td>40</td>
<td>1.312e-4</td>
<td>3.754</td>
<td>39.694</td>
<td>28.492</td>
<td>12.95</td>
</tr>
<tr>
<td>L1</td>
<td>WA</td>
<td>40</td>
<td>1.216e-4</td>
<td>3.817</td>
<td>42.474</td>
<td>16.731</td>
<td>12.73</td>
</tr>
<tr>
<td>BSBL</td>
<td>FT</td>
<td>60</td>
<td>1.369e-4</td>
<td>3.534</td>
<td>43.489</td>
<td>55.479</td>
<td>7</td>
</tr>
<tr>
<td>BSBL</td>
<td>DCT</td>
<td>60</td>
<td>9.538e-5</td>
<td>3.455</td>
<td>42.839</td>
<td>41.864</td>
<td>7</td>
</tr>
<tr>
<td>BSBL</td>
<td>WA</td>
<td>60</td>
<td>1.681e-4</td>
<td>3.045</td>
<td>44.391</td>
<td>38.454</td>
<td>7</td>
</tr>
</tbody>
</table>

CPU = central processing unit; SR = subsampling rate; CS = compressive sensing; MAE = mean absolute error; SNRe = signal-to-noise ratio; CNRe = contrast-to-noise ratio; FT = Fourier transform; DCT = discrete cosine transform; WA = wave atom; BSBL = block sparse Bayesian learning.

Figure 13. (a) SNRe and (b) CNRe of the elastograms as a function of subsampling rate. The image quality measures are computed on the elastograms produced from the original data and from various CS reconstruction frameworks. SNRe = signal-to-noise ratio; CNRe = contrast-to-noise ratio; CS = compressive sensing; FT = Fourier transform; DCT = discrete cosine transform; WA = wave atom; BSBL = block sparse Bayesian learning.

The results indicate that BSBL-based CS frameworks generally delivered the superior performance to L1-based ones. Particularly, the CS framework adopting BSBL-DCT combination yielded the lowest MAE and the highest SNRe and CNRe among all combinations, and possibly the optimal CS reconstruction framework for elastography. The results also suggest that the maximum data reduction (subsampling) rate for generating reasonable elastograms is 60% for BSBL-DCT framework.

Future work will consist of extending the CS reconstruction to allow real-time computation. Currently, the computation for CS reconstruction is so heavy that real-time processing is difficult to achieve. Improving the algorithm for efficient and fast computation is essential to the application of CS to portable ultrasound. Another important improvement involves investigating other reconstruction algorithms and model bases, specifically adapted to ultrasound RF data. Such improvement would make it possible to build an even sparser
representation than current BSBL-DCT combination, thus allows better reconstruction for a given subsampling rate.

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