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| Keywords: | mechanical properties, strain, stress, tension, uniaxial |
The Application of Digital Image Techniques to Determine the Large Stress-Strain Behaviors of Soft Materials

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ABSRACT

Understanding the mechanical properties of soft materials such as stress-strain behavior over a large deformation domain is essential for both mechanical and biological applications. Conventional measurement methods have limited access to these properties because of the difficulties in accurately measuring large deformations of soft materials. In this study, we optimized digital image correlation (DIC) method to measure the large-strain deformations by considering referencing scheme and frame rate. The optimized DIC was applied to characterize the stress-strain behaviour of a polydimethylsiloxane (PDMS) elastomer as a model soft material. A series of comparative experimental studies and finite element analysis were performed; they indicated the advantages of optimized DIC over conventional methods such as robustness to slip, insensitivity to boundary conditions and the ability to yield consistent and reliable results. These advantages enabled the optimized DIC to perform an in-depth analysis of the behaviour of soft materials at large strain domain. An empirical constitutive equation to describe the large stress-strain behaviours of PDMS was proposed and verified by finite element simulations that shows excellent agreements with experimental results.

Key words: Soft materials, PDMS, stress-strain curve, digital image correlation, large strain behaviours
INTRODUCTION

Soft materials such as elastomers, hydrogels and biological tissues have much more complex behaviours and are less understood than pure solids and liquids, but play increasingly important roles in biomedical engineering and micro to nano scale technologies [1-5]. For instance, polyacrylamide chemical gels are employed as the substrate in cell mechanics studies [6], while silicone rubbers are used as implantable cosmetic reconstructive materials due to their biocompatibility and tissue-like mechanical properties [7]. One of the applications in which reliable mechanical properties might be critical is the injection of bio-polymer based hydrogels into the highly stressed environment of the heart wall to ward off end stage heart failure [8].

Accurate strain measurement in a large deformation region is particularly challenging for soft materials. Standard tensile test schemes [9, 10] use dumbbell shaped specimens. These minimize the effect of grip region tri-axial stress state observed in stress-strain results generated with straight (strip) specimens [11], but typically necessitate the use of contact type sensors to isolate gage section response from the overall deformation. Unfortunately the stiffness of contact type sensors prevents their applications to soft materials. On the assumption that deformation primarily takes place in the gauge section, gauge response is frequently approximated by overall elongation [12-14]. However, at high strains deformation outside the gauge section becomes considerable which makes this approach inaccurate [14]. Recognizing this, some researchers introduced a constant correction factor determined by manual measurement or FEM (finite element method) simulation, to convert the overall strain to the gauge section strain [14]. Use of a constant correction factor is valid at small strains or when a constant ratio is maintained between strains inside and outside of the gauge section; however, the
nonlinear stress-strain relationships common in soft materials result in strain ratios that are functions of elongation. Another confounding factor when gage length elongation is not directly measured is the slip between the sample and the grips. Self tightening grips may not respond properly to specimens below certain stiffness while fixed grips cannot respond to the thickness reduction induced by an axial deformation.

To directly measure the gauge section strains, non-contact sensors such as video and laser extensometers are used. Analyzing video data from a tensile test using digital image cross correlation (DIC) [15-17] is one of the most popular methods since it can measure the strain field in a large domain. This method tracks the movement of multiple points on the sample surface by comparing images from different deformed states to a reference image. Using relative displacements of the points the complete strain field can be estimated with sub-pixel accuracy [17, 18].

Two types of referencing schemes are possible. Under fixed referencing an image from the undeformed state is used as a reference image [19]. This scheme is not susceptible to accumulated error as all comparisons are made back to the undeformed state; however, when specimen deformation becomes severe the difference between images may prevent accurate results from being obtained. Dynamic referencing overcomes this difficulty by using the previous deformed state as a reference but at the cost of allowing the potential for accumulated error. Many studies, e.g. [15-20], have used DIC to characterize mechanical properties of materials; however, few studies published in the open literature have considered the effect of referencing scheme on the performance of DIC. This may be partly because the commercial codes utilized do not allow dynamic referencing [21].

This study used polydimethylsiloxane (PDMS) elastomer as a model soft material due to its
purely elastic behaviour and the ease of fabrication. There have been a few studies applying DIC to PDMS under several deformation modes, (Berfield et. al. [15]: under tensile, Nunes [16]: under shear); however, these studies were limited to the small deformation regime. This study investigated the performance of various testing methods including DIC by applying them to large deformation of PDMS, and validated the results with virtual test procedure. We expect that the testing methods proposed in this study can be applied to characterize the large deformation behavior of other soft materials.

EXPERIMENTAL

Specimen Preparation

PDMS was prepared from a two-component kit (Sylgard Elastomer 184, Dow Corning Corporation, Midland, MI). The base and cross-linker were mixed at a ratio of 10:1 for 10 minutes, degassed in a desiccator for 15 minutes and cast into a Teflon mold. Samples were cured in an oven at 90 °C for 90 minutes under an unconfined condition, i.e. the mold was not capped during curing. Dumbbell and strip specimens with geometries shown in Fig. 1(a) were produced.

The DIC method requires proper patterning on the specimen surface. Random speckle patterns were generated by spraying opaque black paint (Createx) with an airbrush (Eclipse HP-CS, Iwata). The airbrush has a 0.35 mm nozzle diameter, allowing for an excellent paint diffusion and high resolution of tiny paint droplets (speckles) as shown in Fig. 1(b). It was verified in preliminary tests that the surface patterning did not affect the mechanical properties of the specimen.
Test Setup

Tensile tests were performed using a TA material testing machine (TA.xt Plus, Stable Micro Systems, New Jersey) with a 5 kgf load cell. During the tests, force, displacement, and time were recorded by the computer while specimen images were captured by a high resolution CCD camera (1028x1008 pixels, STC-CL202A, SENTECH) through a camera link (NI PCIe-1427, National Instrument). The CCD camera with a 25mm manual focus iris lens was placed 70 mm away from the specimen, resulting in a spatial resolution of 56.06 pixels/mm. Mechanical testing and image capture were synchronized and controlled by custom software (LabVIEW V8.5, National Instrument).

Tensile Tests

Both dumbell and strip type specimens were used for tensile tests. To determine the influence of strip specimen aspect ratio, the length to width ratio was varied between 1 and 15, (L₀/W in Fig. 1(a)).

Cyclic tensile tests, consisting of five loading-unloading cycles, were used to examine the error accumulation and grip slippage. In each cycle, tensile loading was applied until the engineering stress reached 2 MPa. Unloading continued until the cross-head returned to its initial position. Between cycles any residual compressive stress or change in specimen shape was noted and released by moving the cross-head until the load became zero. Ultimate tensile tests were also conducted to determine the overall stress-strain curve and failure point. A constant crosshead speed of 12mm/min was used for all tests (loading and unloading).

METHODS
**Digital Image Correlation**

Among the various image tracking algorithms such as cross-correlation [18], gradient descent search [22], snake method [23], and sum of squared differences [24], the fast normalized cross-correlation (FNCC) algorithm [25] was adopted because of its computational efficiency and robustness to lighting conditions [26]. FNCC algorithm was implemented in internally developed Matlab DIC code.

**Stress / Strain Calculation**

Engineering stress, $\sigma_E$, and strain, $\varepsilon_E$, were calculated using the following relationships,

$$\sigma_E = \frac{F}{A_0} \quad \text{and} \quad \varepsilon_E = \frac{\Delta L}{L_0}$$

where $F$ is the force measured by the load cell, $A_0$ initial cross-sectional area, $L_0$ original length and $\Delta L$ elongation. Strains were estimated in two ways. In conventional testing schemes (CS) using overall elongation (10-12), crosshead displacement was measured to be used as $\Delta L$ in Eq. (1). In the tests using dumbbell specimens, $L_G$ in Fig. 1(a) was taken as $L_0$, while initial distance between the grips was regarded as $L_0$ for strip specimens. In DIC method (DIC), nine rectangular grid points were chosen in the middle of the gauge section following ASTM standard [27] to be tracked by the algorithm, as shown in Fig. 1(b). The engineering strain was calculated by averaging the strains of those points as:

$$\varepsilon_E = \frac{\Delta L}{L_0} = \frac{(y_3 + y_6 + y_9 - y_1 - y_4 - y_7) - (Y_3 + Y_6 + Y_9 - Y_1 - Y_4 - Y_7)}{(Y_3 + Y_6 + Y_9 - Y_1 - Y_4 - Y_7)}$$

(2)

where $Y_i$ are the y-directional coordinates of grid points on the undeformed image, and $y_i$ y-coordinates of the same grid points on the deformed images tracked by DIC. True strains in x- and y- axis directions were also calculated from the displacements of grid points as
\[
(e_x)_T = \ln \left( \frac{L_f}{L_0} \right) = \ln \left( \frac{x_3 + x_6 + x_9 - x_1 - x_4 - x_7}{X_3 + X_6 + X_9 - X_1 - X_4 - X_7} \right)
\]

\[
(e_y)_T = \ln \left( \frac{L_f}{L_0} \right) = \ln \left( \frac{y_3 + y_6 + y_9 - y_1 - y_4 - y_7}{Y_3 + Y_6 + Y_9 - Y_1 - Y_4 - Y_7} \right)
\]

Note that displacement rate (i.e. infinitesimal strains, \( \varepsilon_{ij} = 0.5(\partial u_i/\partial x_j + \partial u_j/\partial x_i) \)) are commonly adopted for strains in DIC applications; however, this is not valid in large deformations.

By assuming a plane stress condition in the gauge section, Poisson’s ratio \( \nu \) was determined by using Eq. (3):

\[
\nu = \frac{\varepsilon_x}{\varepsilon_y}
\]

**DIC Optimization**

The effect of referencing scheme on DIC performance was investigated by applying fixed and dynamic referencing schemes to a simple tensile test. Images were taken during the test using the dumbbell specimen elongated up to 70%. The images were analyzed using both schemes.

The effect of frame rate using dynamic referencing was examined by a simple translation test. In this test a speckle patterned glass plate clamped in the TA testing machine was moved vertically upward at a speed of 20 \( \mu \)m/s while pictures were taken at 5 frames per second. Different frame rates of 2.5, 1.67, 1.25, 1, 0.5, 0.25, 0.125, and 0.066 fps were achieved by skipping images at fixed intervals in the DIC analysis to simulate a range of frame rates. The corresponding displacements of the reference image were 0.112, 0.224, 0.449, 0.676, 0.897, 1.121, 2.24, 4.49, 8.97, and 17.94 pixels per frame. Error was determined by comparing the
average displacement calculated for nine points to the known displacement.

**Virtual Tensile Test**

To estimate the accuracies of the stress-strain curves determined by various testing methods, tensile tests were simulated using FEM. In this simulation, load-displacement curves were produced by 3-D FEM model (ABAQUS 6.5 Standard) of dumbbell specimen consisting of 1024 20-node quadratic brick elements, employing each stress-strain curve as a material property. Considering the geometrical symmetry of the specimens, only half quarters of the specimens were modeled. The empirical constitutive equation obtained through curve-fitting to experimental stress-strain curve was coded into the FEM model using UMAT that is a user-defined module in ABAQUS for material properties.

Since the empirical constitutive equation is highly nonlinear, the stress-strain relationship based on linear elasticity (Hooke’s law) cannot be used for UMAT. Also other nonlinear elastic stress-strain functions provided by ABAQUS such as hyperelasticity cannot be matched with the stress-strain curve. Therefore, new stress-strain relations based on the proposed constitutive equation should be defined. For this, experimentally determined uniaxial stress-strain curve was regarded as effective stress-strain curve, and assumed to be the stress function for the deformation. For monotonically increasing loading, nonlinear elastic deformation and plastic deformation cannot be discerned; therefore, the incremental deformation theory of plasticity [28] could be invoked as:
\[ \bar{\sigma} = f(\bar{\varepsilon}) \]

\[ d\varepsilon_x = \frac{d\bar{\varepsilon}}{d\bar{\sigma}} \left[ d\sigma_x - \nu (d\sigma_y + d\sigma_z) \right] \]

\[ d\varepsilon_y = \frac{d\bar{\varepsilon}}{d\bar{\sigma}} \left[ d\sigma_y - \nu (d\sigma_z + d\sigma_x) \right] \]

\[ d\varepsilon_z = \frac{d\bar{\varepsilon}}{d\bar{\sigma}} \left[ d\sigma_z - \nu (d\sigma_x + d\sigma_y) \right] \]

\[ d\gamma_{xy} = 2(1 + \nu) \frac{d\bar{\varepsilon}}{d\bar{\sigma}} d\tau_{xy} \]

\[ d\gamma_{yz} = 2(1 + \nu) \frac{d\bar{\varepsilon}}{d\bar{\sigma}} d\tau_{yz} \]

\[ d\gamma_{zx} = 2(1 + \nu) \frac{d\bar{\varepsilon}}{d\bar{\sigma}} d\tau_{zx} \]

where \( \bar{\sigma} = f(\bar{\varepsilon}) \) is the newly proposed constitutive equation where \( \bar{\sigma} \) and \( \bar{\varepsilon} \) are the effective stress and effective strain, respectively. \( d\varepsilon_{ij} \) and \( d\gamma_{ij} \) are the strain increment tensors at each step.

**RESULTS AND DISCUSSION**

**DIC Optimization**

**Referencing Optimization**

In Fig. 2 the abilities of fixed and dynamic referencing to track the progress of specimen deformation are qualitatively compared. When the deformation is small, both schemes yield almost the same results; however as the deformation becomes significant, fixed referencing cannot follow the movements of the grid points (Fig 2(a)), while the dynamic referencing consistently demonstrated excellent tracking capability (Fig. 2(b)).

For a quantitative comparison between the two referencing schemes the average NCC
values, degree of matching for the 9 grid points in Fig. 2(a) and 2(b) are plotted in Fig. 3. While the NCC value for dynamic referencing is very consistent in the vicinity of 0.9 out of a possible 1.0; a steady decline and increasing scatter for the fixed referencing scheme indicate a decreasing ability to identify the new location of a point with increasing deformation. When the strain exceeds 0.13 the NCC value drops below the threshold of 0.5 indicating that the result is unreliable.

**Frame Rate Optimization**

The effect of frame rate on DIC performance adopting dynamic referencing is presented in Fig. 4, using the pixel movement per frame as the abscissa, and as the ordinate the estimated average displacement of 9 grid points for the known displacement of 9.95mm. It can be seen that though the average is always within one standard deviation of the real displacement, acceptable accuracy is only obtained when the movement per frame is greater than one pixel. It is also observed that displacements per frame larger than one pixel do not have any significant influence on accuracy, with the error in each case being in the order of 1%. Even with the sub-pixel algorithm using quadratic interpolation, movements less than one pixel cannot be accurately estimated. This should be caused by the discontinuous nature of DIC data. If the frame rate is high so that multiple estimated movements of less than one pixel are added together under dynamic referencing, these errors are accumulated to lead the incorrect results. Conversely, error in estimating movements greater than one pixel appears to be truly random allowing it to cancel itself out as multiple results are summed together for the final displacement. Note that the error due to sub-pixel movement is not accumulated in fixed referencing.

Based on the above results the dynamic referencing scheme with frame rate adjusted to a minimum displacement of one pixel per frame between two consecutive images was employed.
for the primary analysis.

**Cyclic Tensile Tests on Dumbbell Specimen**

Fig. 5 shows engineering stress-engineering strain curves from 5 cycles of loading-unloading tensile tests performed on a dumbbell specimen, evaluated by conventional scheme (CS) and DIC method. Loading and unloading branches are indicated by upward and downward arrows, respectively. It is notable that the stress-strain curve in the first cycle evaluated by CS (CS-1 in Fig. 5) deviates from other CS curves (CS-2~5), and a large hysteresis loop was formed by the loading and unloading branches. An examination of the specimen images before and after the first cycle, Fig. 6(a) and 6(b) respectively, suggests that significant slip occurred in this cycle (note un-patterned region exposed by specimen pullout). No amount of tightening prevented this slip from occurring; however, this slip occurred only once in the first cycle as the specimen settled in the grips.

The high degree of consistency observed in loading/unloading curves after the first cycle, CS-2 to 5 in Fig 5, suggests no irreversible process occurred during these tests. A small amount of hysteresis was observed which might be attributable to material viscoelastisty or some other dissipative process.

Little or no hysteresis was observed in DIC results which are all in good agreement with one another and fall onto a single curve (DIC-1~5). This suggests that DIC method is highly robust to slip and can yield consistent result irrespective of slip.

It is interesting to note that the DIC stress-strain curves are significantly different from those given by CS-2~5, with the stress level in DIC curves more than twice of that in CS at the same strain. It is known that the CS overestimates the strain due to the deformation outside the gauge section [13, 14]. Schneider et al. [14] multiplied the measured strain by a correction factor of m
= 0.49~0.50 to convert the overall strain to gauge section strain. Following this scheme CS-C in Fig. 5 was generated. While it is much closer to the DIC curve, differences still exist in magnitude of stress and the level of hysteresis loop. Adjusting correction factor might improve matters further but cannot completely resolve the difference.

The difference was further investigated to estimate the reliabilities of CS and DIC based measurements.

**Hysteresis Analysis**

Hysteresis loop in cyclic stress-strain curve is one of the properties typifying visco-elastic materials. Other visco-elastic properties include stress relaxation and creep. To verify the visco-elastic properties of tested PDMS, specimens were loaded at constant strain and at constant stress, respectively, for 24 hours to investigate the stress relaxation and creep behaviors. The test results (not included in this paper) show that stress relaxation or creep did not occur in the tested PDMS. This is consistent with the previous reports suggesting that fully cured PDMS is purely elastic at room temperature [29, 30]. Therefore, we concluded that the tested PDMS should not have visco-elastic properties and the hysteresis loop observed by CS-C curve must have come from some other cause.

By carefully examining the side view images of the tensile specimens as shown in Fig. 6(c), we noticed that a portion of the specimen pulled out from the grip region under tensile loading (Fig. 6(d)) and retracted to its original position when it was unloaded (Fig. 6(e)), as indicated by the movement of the arrow pointing the same spot. This localized deformation is expected in all materials, but is much more significant in soft materials. It resulted in a changing effective gage length during CS based measurements. The frictional forces between the grip surfaces and sample oppose movement resulting at the same load, in differing effective gage lengths in
the loading and unloading branches, which should have caused the hysteresis loop. The overlapping of the CS-2~5 curves indicated that this type of slip was reversible, which made the detection extremely difficult. On the contrary, DIC measurements guarantee a constant gage length and are made only in the middle of the gage section where the uniaxial stress assumption is most valid and so avoid this difficulty. The fact that this slip in the grips can introduce hysteresis must be considered in studies dedicated to viscoelastic properties, particularly in the large deformation/non-linear region.

**Cyclic Tensile Tests on Strip Specimen**

CS based grip settling behaviour was similar to dumbbell specimen, i.e. existence of irreversible and reversible slips. Because of these slips, the stress strain curve was estimated from the loading portion of the subsequent cycles in Fig. 7. Comparing the stress-strain curves for subsequent cycles, a trend of increasing apparent stiffness with increasing $L_0/W$ is observed. This trend becomes less prominent as $L_0/W$ ratio becomes higher, and the curves superimpose at $L_0/W$ ratios above about 15. The variation of stress-strain curve with $L_0/W$ can be explained by the variation of stress state within the strip specimen. Because of the constraint by gripping, the stress state in the vicinity of end grip is close to triaxial, which changes gradually to uniaxial as the distance from the end grip increases. The size of the triaxial region in the influence of the end grip is independent of the strip length. Therefore, the increase of $L_0/W$ causes the decrease of the ratio of the material volume affected by triaxial stress state, and the measured stress-strain curve approaches the one from purely uniaxial stress state. Since the stress-strain curves in Fig. 7 converge to the curve for $L_0/W =15$, it should be very close to the uniaxial stress-strain curve. The insert of Fig. 7 shows both of loading and unloading portions for SS – 1 and 15 indicating that the observed hysteresis is inversely proportional to $L_0/W$. This bears out our previous
speculation that it is grip induced rather than inherent to the material. DIC results are again highly consistent for all \( L_0/W \) ratios tested (DIC-SS in Fig. 7) and in almost perfect agreement with the dumbbell results (DIC-Dumbbell). Therefore, it can be stated that optimized DIC can yield the stress-strain curve independent of specimen geometry. The similarity between the DIC curve and the one for \( L_0/W=15 \) suggests that CS with \( L_0/W\geq15 \) may be adequate if extreme accuracy is not necessary.

**Ultimate Tensile Tests**

The results from ultimate tensile tests were analyzed by DIC and Schneider et al.’s corrected crosshead displacement scheme [14], CS-C. In Fig. 8 they are compared to the results obtained by Khanafer and coworkers [31] for the same material using crosshead displacement. Note that Ref. [31] proposed a 3\(^{rd}\) order polynomial equation for the curve fitting. The curve from strip specimen for \( L_0/W=15 \) were converted into true stress-true strain curve using Eq. (3) and also plotted (SS) in Fig. 8. Poisson’s ratio was required for this conversion of engineering stress into true stress. From axial and transverse true strains measured by DIC and Eq. (4), Poisson’s ratio was determined to be 0.5±0.03 which is consistent with the common belief that fully cross-linked PDMS is be incompressible [32, 33]. From this result, Poisson’s ratio was assumed to be 0.5.

All four curves overlap in small strain region, showing almost linear increasing trend up to around 0.2. They start rising exponentially beyond that strain, indicating that strain-hardening becomes more significant with the progress of deformation. Different hardening behaviors are demonstrated by different curves, with DIC being most significant followed by SS, CS-C, and the curve from Ref. [31]. DIC and SS curves look very similar; however, the magnified view in the insert shows that there is a difference.
Note that the severe strain hardening behaviors in DIC or SS curves in Fig. 8 cannot be fitted by the 3rd order polynomial proposed by Ref. [31]. We had also attempted to use common constitutive models such as rubber elasticity [34], Mooney and Rivlin [35], BST equation [36], G’Sell and Jonas [37] to describe the observed non-linear behaviours. However, none of them provided good agreements except the BST equation, which has a very complex form with 4 fitting parameters. The resulting fitting parameters in BST equation do not have any intrinsic meaning, and it is difficult to make a link to the deformation mechanism [38].

Based on the observed strong strain-hardening behaviour with an almost vertical asymptote at large strain, we proposed the following form of constitutive equation to describe the stress-strain behaviour across large strain region:

$$\sigma_T = E \left[ \varepsilon_T + \frac{\varepsilon_T^A}{B - \varepsilon_T} \right]$$  \hspace{1cm} (6)

where $E$ is elastic modulus; $A$ and $B$ are two fitting constants related to the strain-hardening behavior. Eq. (6) has enough flexibility to fit all four curves in Fig. 8 almost perfectly using the fitting constants in Table 1 that are evaluated by least square fitting method. The first term in Eq. (6) dominates true stress when strain is small, while the importance of second term increases with the increase of strain. Note that Eq. (6) has the vertical asymptote at $\varepsilon_T = B$, which implies that the stiffness approaches infinity as the strain is getting close to B. The specimen could not be deformed up to asymptotic strain, as the specimen failed before gauge section strain reached that strain because of the stress concentration at the round corner between gauge and nongauge sections.

**Virtual Tensile Test using FEM**

Fig. 9 shows the load-displacement curves from the FEM simulations and the experiment.
The FEM simulation adopting the DIC stress-strain curve shows an excellent agreement with the experimental data (circle), while other simulation results deviate from the experimental data in the similar manner as their stress-strain curves deviate from DIC stress-strain curve. The deviations are noticeable in large strain region; however, the magnified view in small strain region (insert in Fig. 9) also illustrates that simulation result using DIC curve is in much better agreement with the experimental data than any others. The errors between the load-displacement curves from the experiment and simulations are quantified by using the R-squared value for the load-displacement curves, with R-squared value being 1 when two curves are perfectly matched. Table 2 shows that the R-squared value for the simulation curve adopting DIC stress-strain curve is almost 1, while those from other simulations are getting lower than 1 in the order of SS, CS-C and literature. These results indicate that stress-strain curves evaluated by DIC method best represents the real stress-strain behavior and should be very close to actual material property.

**Asymptotic strain-hardening**

The constitutive equation in Eq. (6) suggests strong strain-hardening behaviour caused by both the power law relation in the numerator and the asymptote in the denominator. The power-law equation in the numerator induces comparable degree of strain-hardening as rubber elasticity theory in small strain region. Rubber elasticity assumes that the free energy change in deformation is due to the restraints placed on configurational rearrangement, and is considered to be totally entropic in origin [39]. However, this theory is only applicable to small strains, since the considerable change in the end-to-end distance of the network chains would distort the Gaussian distribution of the statistical elements.

As the strain increases and approaches the asymptotic strain $\varepsilon_T=0.701$, different type of
hardening takes place in a much more significant manner. This strain hardening may be induced by strain crystallization [40, 41], alignment of covalent-bonded polymer chains in the stretching direction. When PDMS is moderately deformed under tension, polymer chains are disentangled and re-oriented to be aligned along the loading direction, which can be represented by the power law relation in the numerator in Eq. (6) or rubber elasticity model. As the strain approaches asymptotic strain, polymer chains are pulled taut that the forces are mostly carried by covalently bonded polymer chains, and the measured stiffness approaches that of polymer chains. Since the covalent bonding is extremely stiff compared to other types of bondings and stiffening mechanisms, the stress-strain curve takes the form of vertical asymptote as presented in Fig. 8.

CONCLUSIONS

This study showed that the selection of reference image has a major influence on the accuracy of DIC results and the adoption of dynamic referencing with a suitable frame rate in DIC analysis can yield much better results than fixed referencing. Optimized DIC method was applied to a large deformation of soft materials. PDMS was used as model soft material and its stress-strain relationship across a large deformation region was evaluated. The comparative study of the stress-strain curves obtained from the conventional tensile test schemes and DIC method suggested that the DIC technique is robust to the slip between the sample and the grips while the accuracy of conventional test scheme is highly affected. The true stress-strain relationship of PDMS in tension evaluated by DIC showed a significant strain-hardening behaviour, with a vertical asymptote at strain $\varepsilon_T = 0.701$. Based on this behaviour a new type of constitutive equation was proposed to account for the significant strain-hardening in large strain
region, and showed excellent agreement with the experimental stress-strain curves. FEM simulation adopting the constitutive equation successfully produced the load-displacement curve showing excellent agreement with experimental data. This suggests that the stress-strain curves determined by the DIC can be used to represent the actual stress-strain behavior of PDMS. Poisson’s ratio of PDMS was also verified to be 0.5 in tension. The optimized DIC technique and analysis method used here may be able to be applied to the studies of other elastomers, gels and biological tissues.

Acknowledgements

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REFERENCES


### TABLES

Table 1  Constants in Eq. (6) for the stress-strain curves from DIC, SS, CS-C, and Ref. (31)

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<th>E</th>
<th>A</th>
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<td>DIC</td>
<td>1.980</td>
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<tr>
<td>SS (L₀/W=15)</td>
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<td>CS-C</td>
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Table 2  R-squared values for the load-displacement curves from experiment and the simulations adopting different stress-strain curves

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<td>$R^2$</td>
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LIST OF FIGURES

Fig. 1. (a) The geometries of the specimens, and (b) the gauge section image under tension on which grid points for DIC analysis are indicated: undeformed (Xi, Yi) and deformed (xi, yi) positions of grid points.

Fig. 2. Qualitative comparison of image tracking abilities: (a) fixed referencing, and (b) dynamic referencing. The engineering strains were measured under dynamic referencing.

Fig. 3. Plots of average normalized correlation coefficients (NCC) and their standard deviations (STD) under fixed and dynamic referencing.

Fig. 4. The dependence of DIC performance on image frame rate. 9.95 mm translation was measured by DIC using the images taken at different frame rates under dynamic referencing.

Fig. 5. Engineering stress-strain curves from cyclic tensile tests on dumbbell specimens measured by conventional scheme (CS), corrected conventional scheme (CS-C), and DIC.

Fig. 6. Front view of patterned specimen before (a), and after (b) the first cyclic loading; Side view of grip region before (c), during (d), and after (e) the subsequent cyclic loadings. The movement of the arrow pointing the same spot of the region indicates that pull-out (d) and retraction (e) occurred in the grip region.

Fig. 7. Engineering stress-strain curves from the cyclic tensile tests on strip specimens.

Fig. 8. True stress-strain curves from DIC, SS, CS-C, and Ref. (31).

Fig. 9. Load-displacement curves from experiment (O) and the FEM simulations adopting constitutive equations for DIC, SS, CS-S, and Ref. (31).
Fig. 1. (a) The geometries of the specimens, and (b) the gauge section image under tension on which grid points for DIC analysis are indicated: undeformed \((X_i, Y_i)\) and deformed \((x_i, y_i)\) positions of grid points.

130x65mm (150 x 150 DPI)
Fig. 2. Qualitative comparison of image tracking abilities: (a) fixed referencing, and (b) dynamic referencing. The engineering strains were measured under dynamic referencing.
Fig. 3. Plots of average normalized correlation coefficients (NCC) and their standard deviations (STD) under fixed and dynamic referencing.
82x62mm (150 x 150 DPI)
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